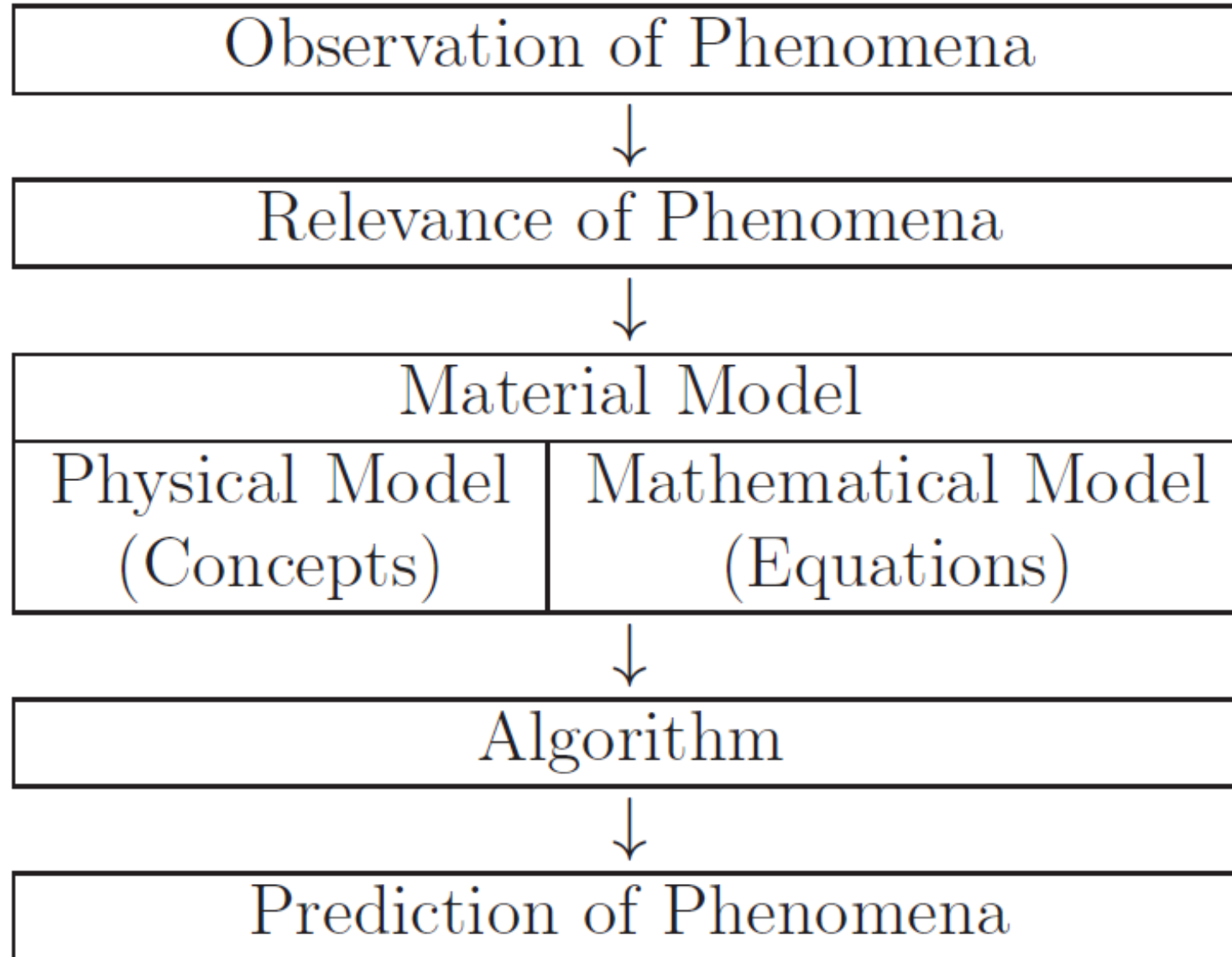
The background is a dark grey surface covered with faint, white, handwritten-style mathematical formulas and mechanical terms. At the top left, 'yield function' is written with an arrow pointing to a formula for  $\Phi$ . Below it, 'coaxial' and 'perpendicular' are written with arrows pointing to different parts of the formula. To the right, 'linear hardening' is written with an arrow pointing to a term in the formula. The formula for  $\Phi$  is  $\Phi = |\sigma| [ \omega |\hat{\epsilon}_n| + [1-\omega] |\hat{\epsilon}_t| ] - [\sigma_y + K\alpha] \langle 0$ . Below this, there are more formulas:  $\dot{\epsilon} = \dot{\gamma} \frac{\partial \Phi}{\partial \sigma} = \dot{\gamma} \text{Sign}(\sigma) [ \omega |\hat{\epsilon}_n| + [1-\omega] |\hat{\epsilon}_t| ]$  and  $\epsilon = \frac{|\sigma|}{K} - 1$ . At the bottom, there are more formulas:  $\hat{\epsilon}_n = \hat{\epsilon} \cdot \hat{P}_n$  and  $\hat{\epsilon}_t = \hat{\epsilon} \cdot \hat{P}_t$ .

# It's the Mechanics, Stupid!

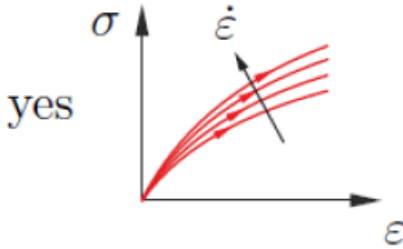
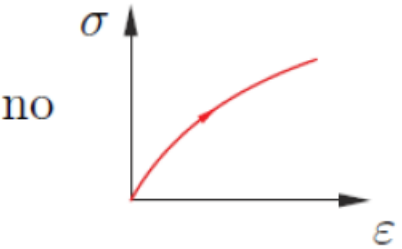
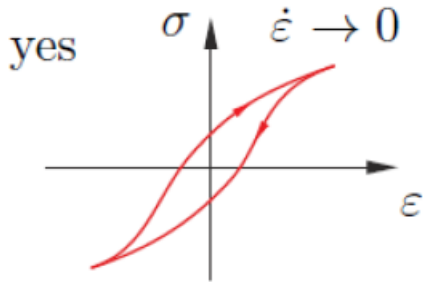
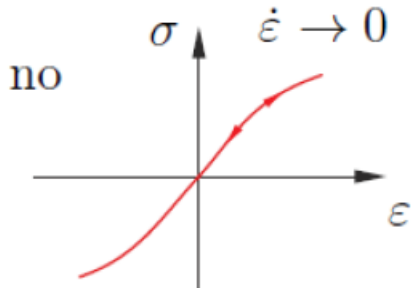
## -The Joy of Material Modelling-

\*modified from Bill Clinton's presidential campaign 1992

# Modelling



# Phenotypes

		rate-dependent behaviour	
			
quasi-static hysteresis		viscoplasticity	plasticity
		viscoelasticity	elasticity

# Phenotypes

Mechanical Phenomena	Rate-Independent	Rate-Dependent
Quasi-Statically Reversible	Elasticity	Visco-Elasticity
Quasi-Statically Irreversible	Plasticity	Visco-Plasticity



# Elasticity

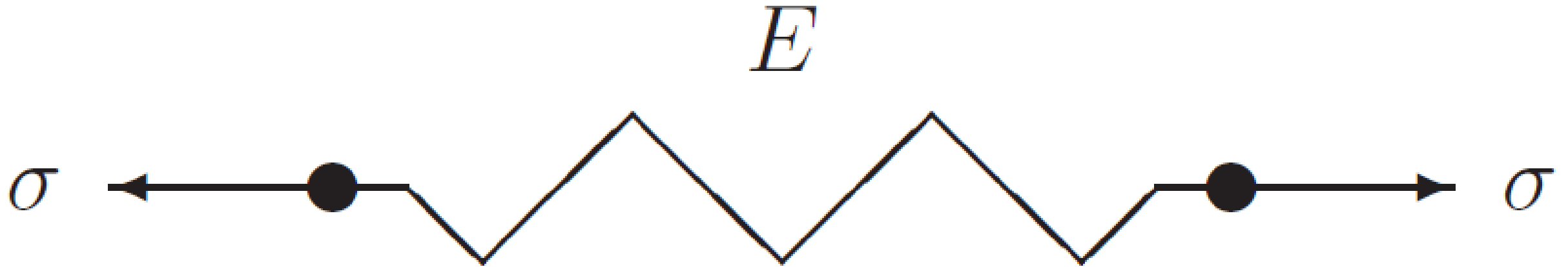


# Hooke Model

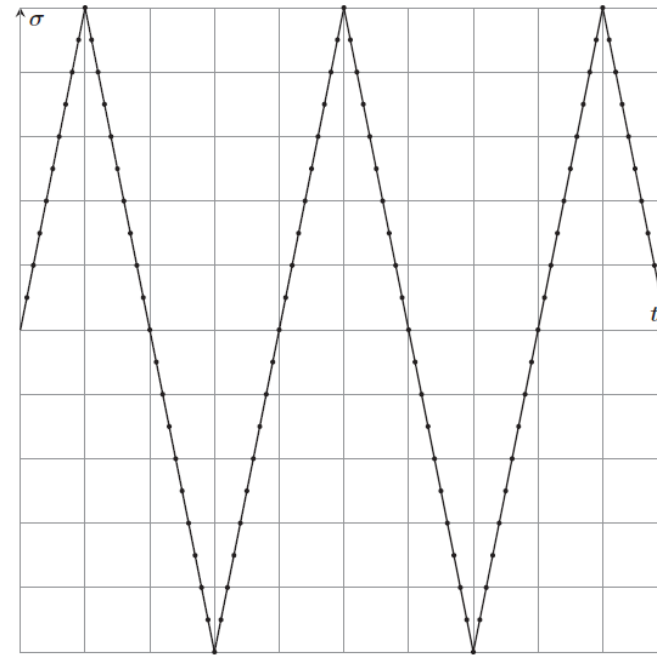
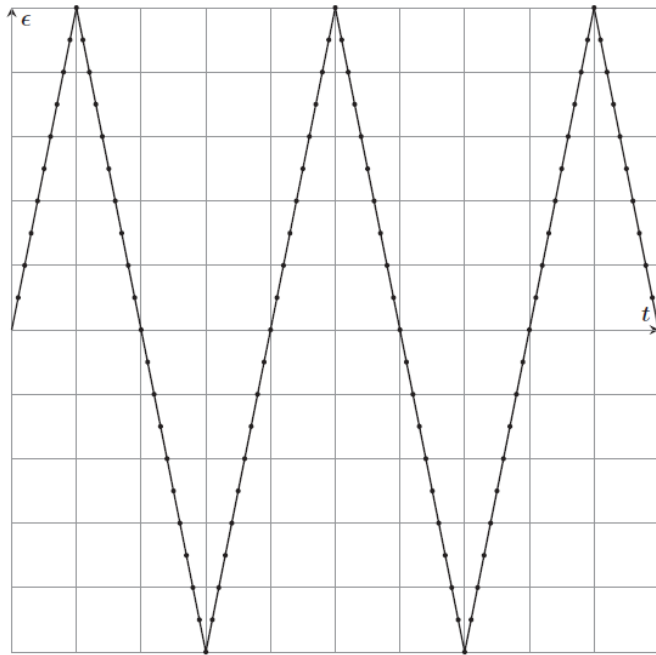
*Robert Hooke* [b. 18.7.1635, Isle of Wight, England, d. 3.3.1703, London, England] was polymath, Curator of Experiments of the Royal Society, Professor of Geometry at the Gresham College in London, Surveyor of the City of London and architect. He largely contributed to mechanics, microscopy, astronomy, meteorology and geology. Due to his tract "De potentia restitutiva" on the elasticity of springs from 1678, the Hooke model of linear elastic solids is named in his honour.



# Hooke Model



# Hooke Model

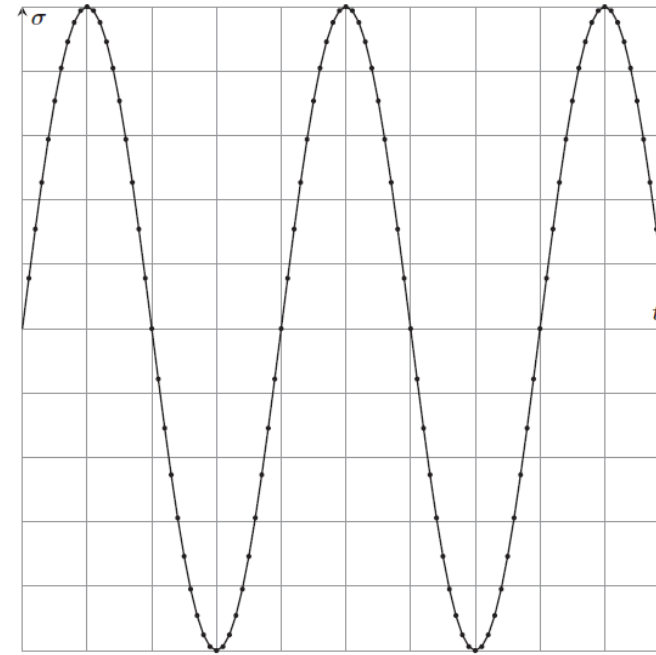
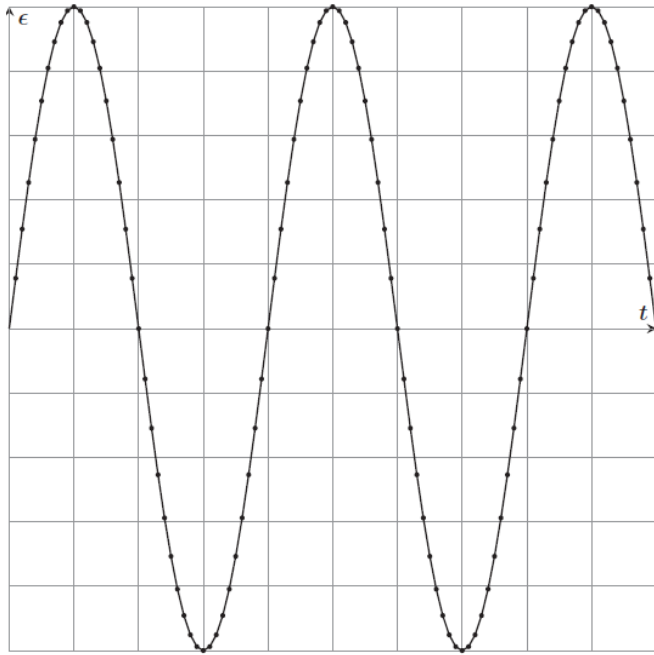


**no phase shift**

**zig-zag strain & stress histories**



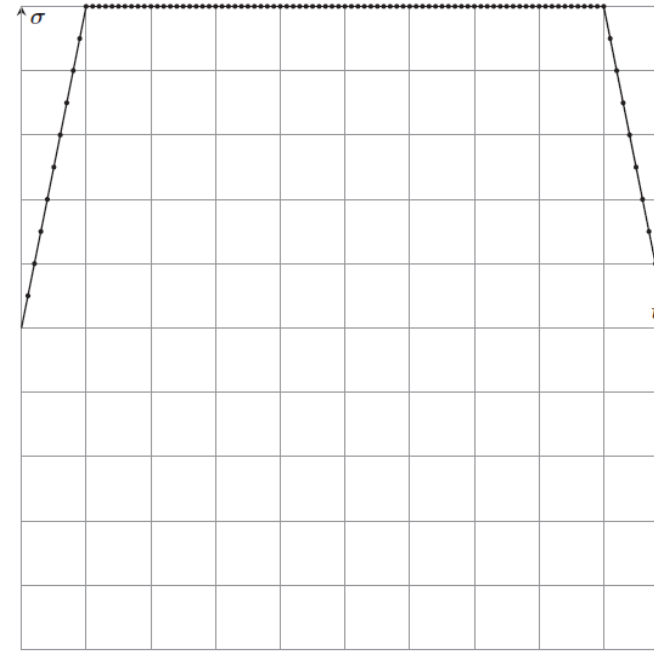
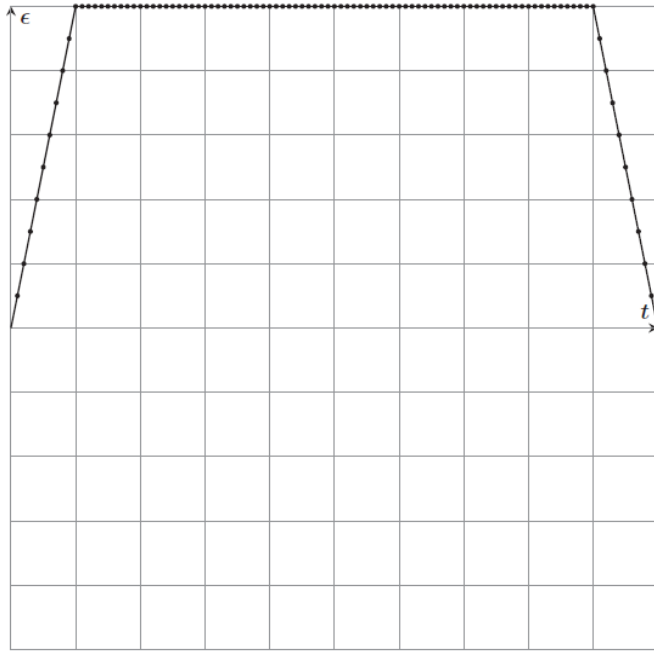
# Hooke Model



**no phase shift**

**sine strain & stress histories**

# Hooke Model



**no creep  
no relaxation  
solid-like**

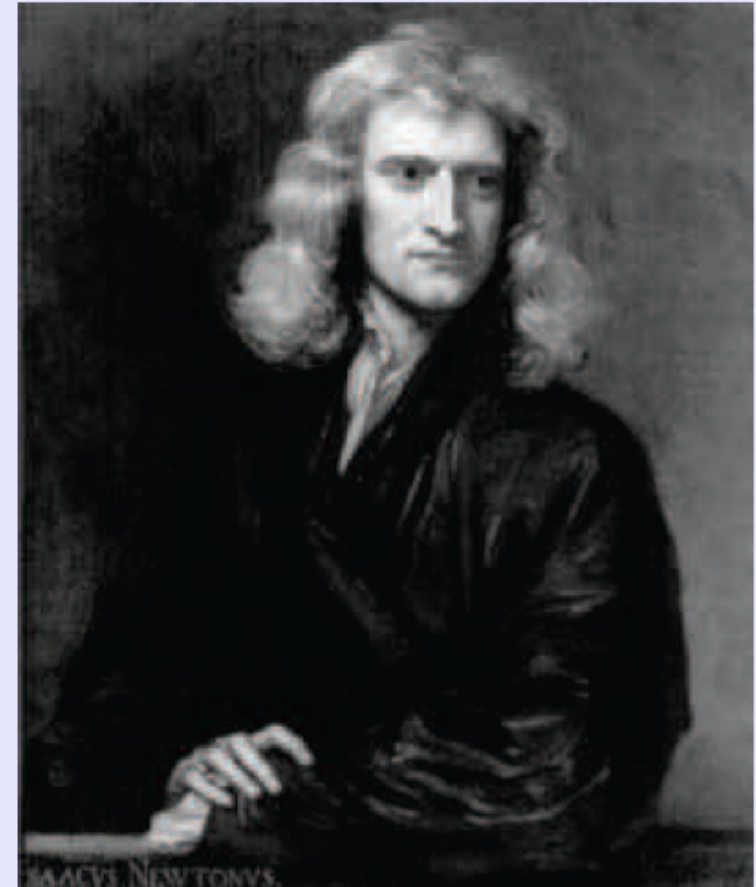
**ramp strain & stress histories**

# Visco-Elasticity



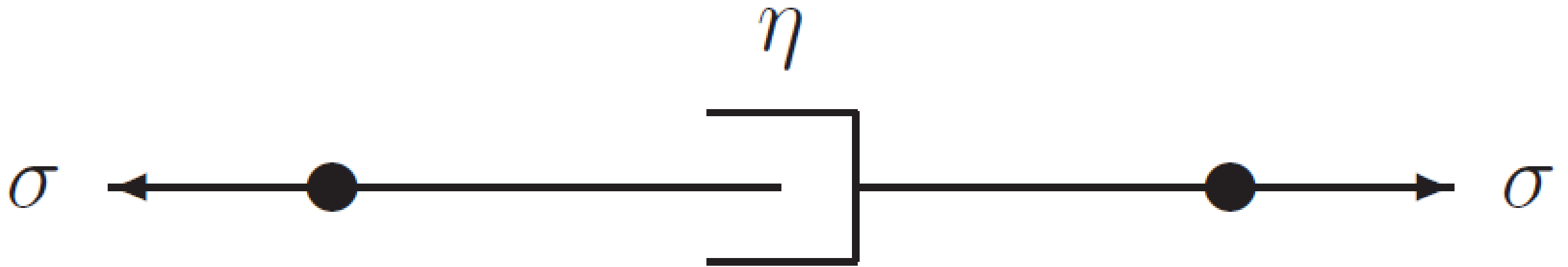
# Newton Model

*Sir Isaac Newton* [b. 4.1.1643, Woolsthorpe, England, d. 31.3.1727, London, England] was Lucasian Professor of Mathematics at the Trinity College in Cambridge. He laid the foundations of classical mechanics in his landmark treatise "Philosophiae Naturalis Principia Mathematica" from 1687. Likewise he is the co-founder of differential and integral calculus (as the opponent to Leibniz) and greatly contributed to optics. The Newton model of viscous fluids is named after him.

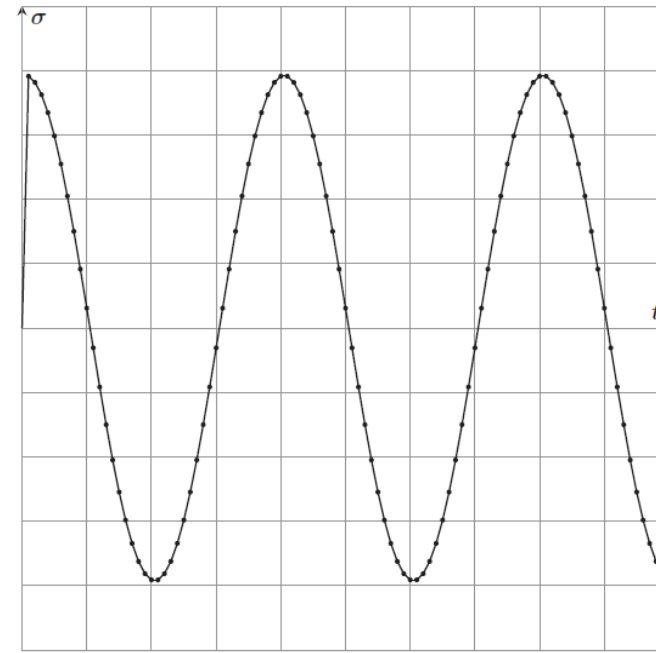
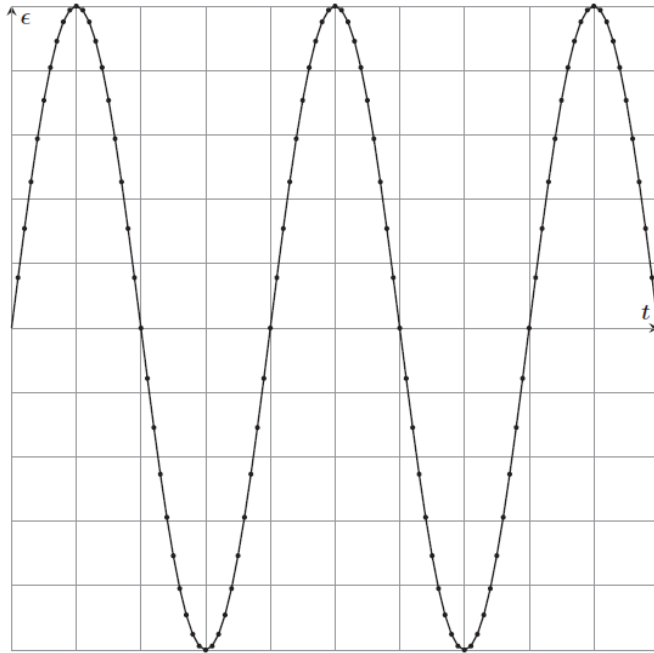




# Newton Model



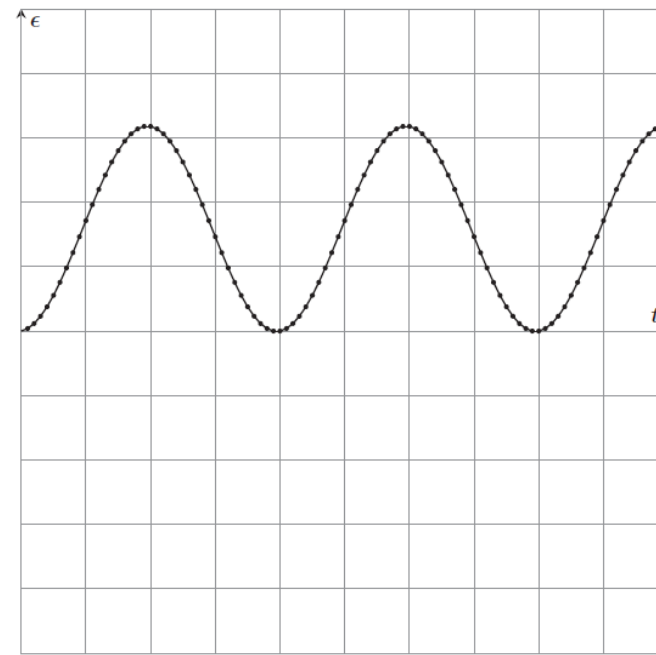
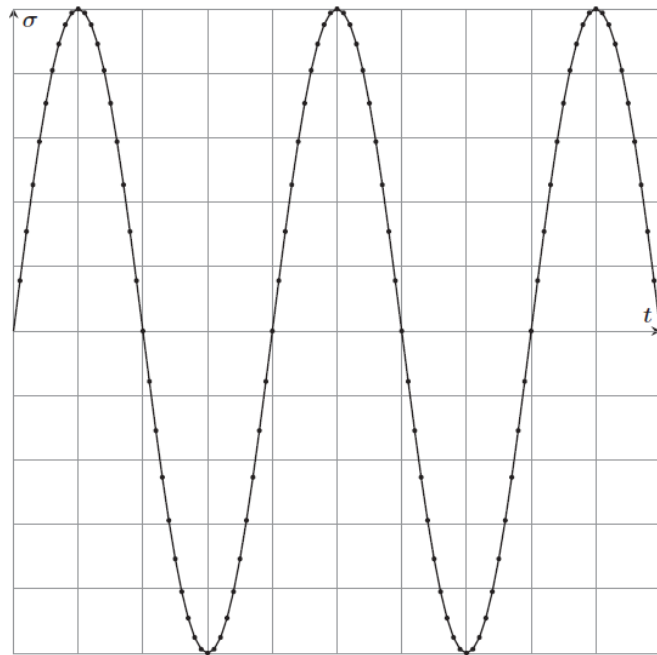
# Newton Model



**phase shift  
fluid-like**

**sine strain history**

# Newton Model

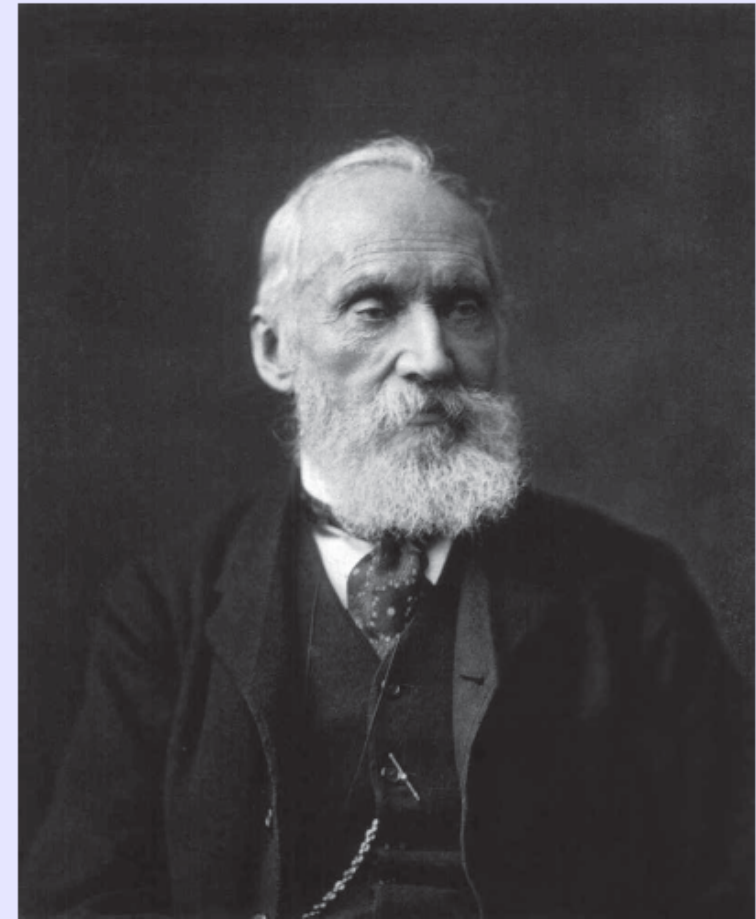


**phase shift  
fluid-like**

**sine stress history**

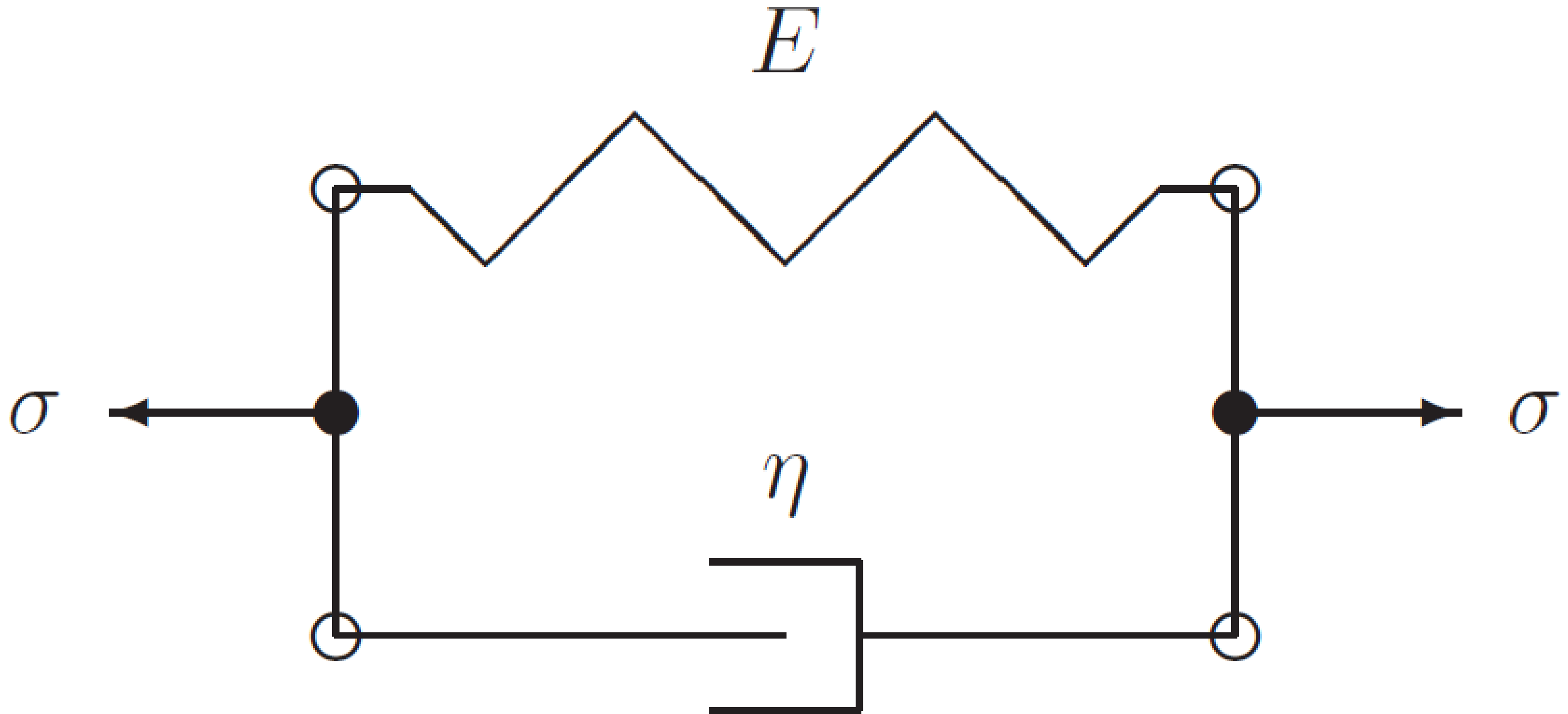
# Kelvin Model

*Sir William Thomson (Lord Kelvin)* [b. 26.6.1824, Belfast, Ireland, d. 17.12.1907, Netherhall, Scotland] was Professor of Theoretical Physics at the University of Glasgow. He worked mainly on thermo- and electrodynamics for which he developed ingenious (measuring) devices. The so-called Kelvin scale is based on his correct determination of the absolute zero temperature. 1892 he became Baron Kelvin in recognition of his achievements in thermodynamics. The Kelvin model for visco-elastic solids is named after him.

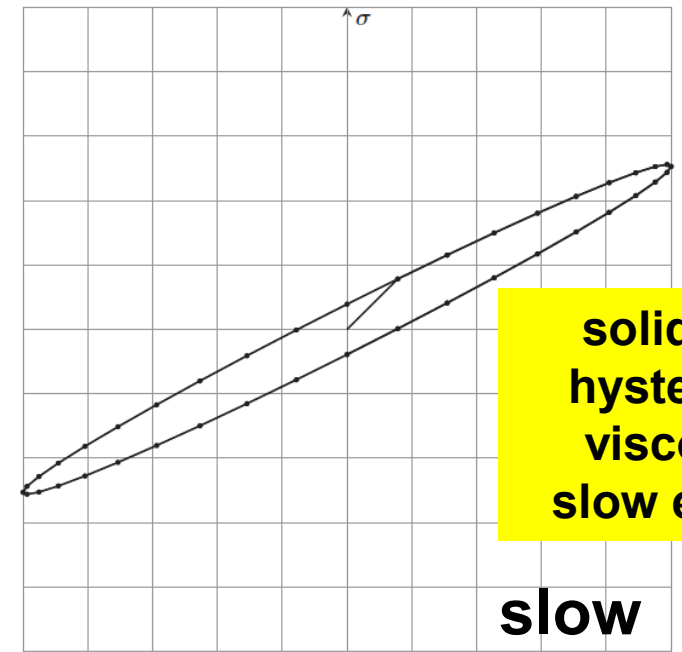
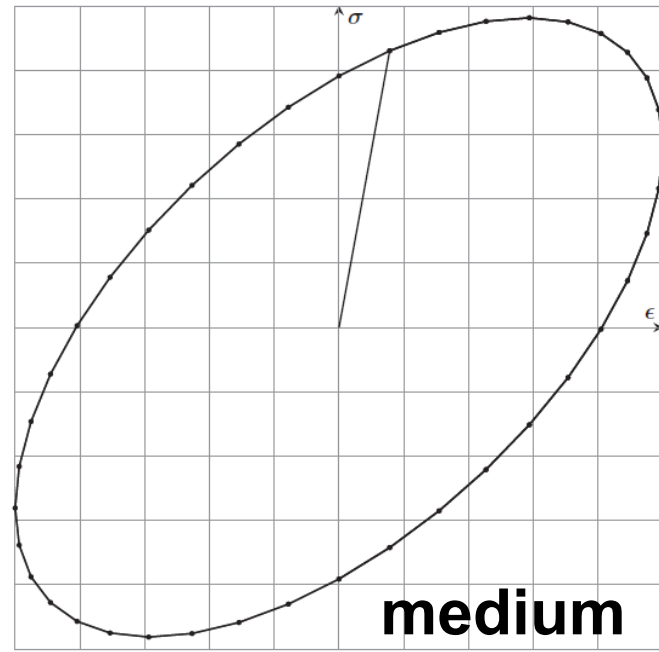
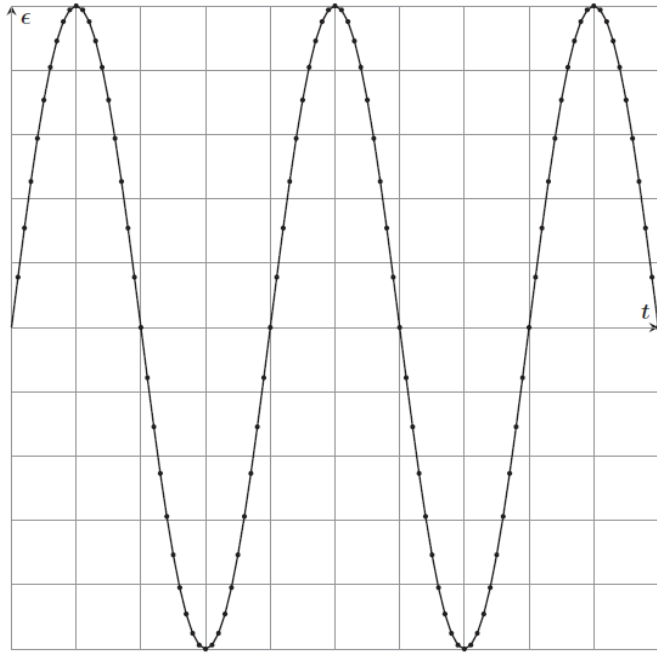




# Kelvin Model



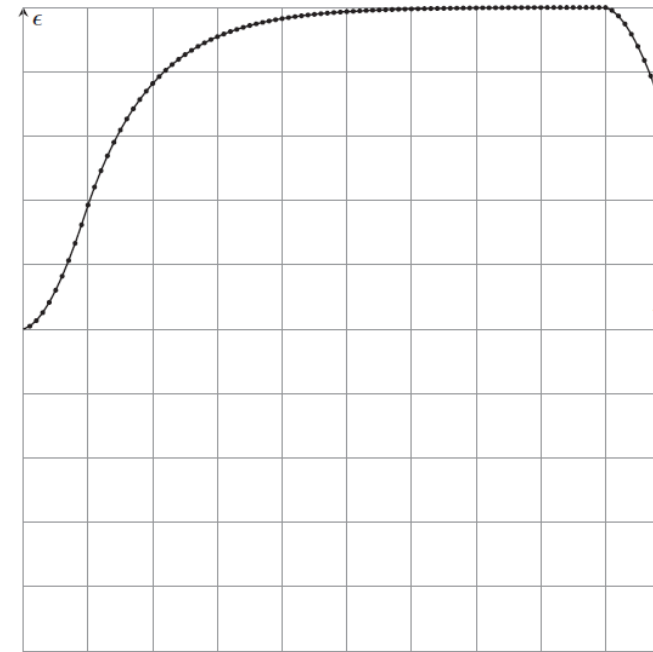
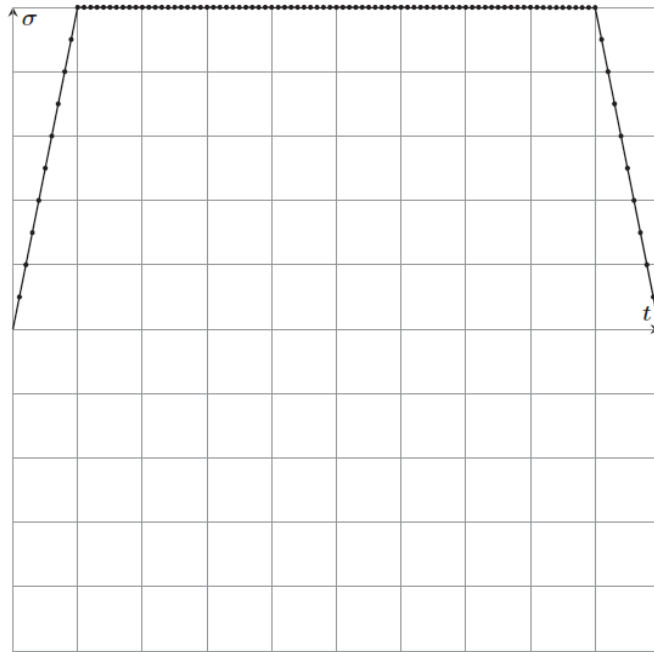
# Kelvin Model



**solid-like  
hysteresis  
viscosity  
slow elastic**

**sine strain history**

# Kelvin Model

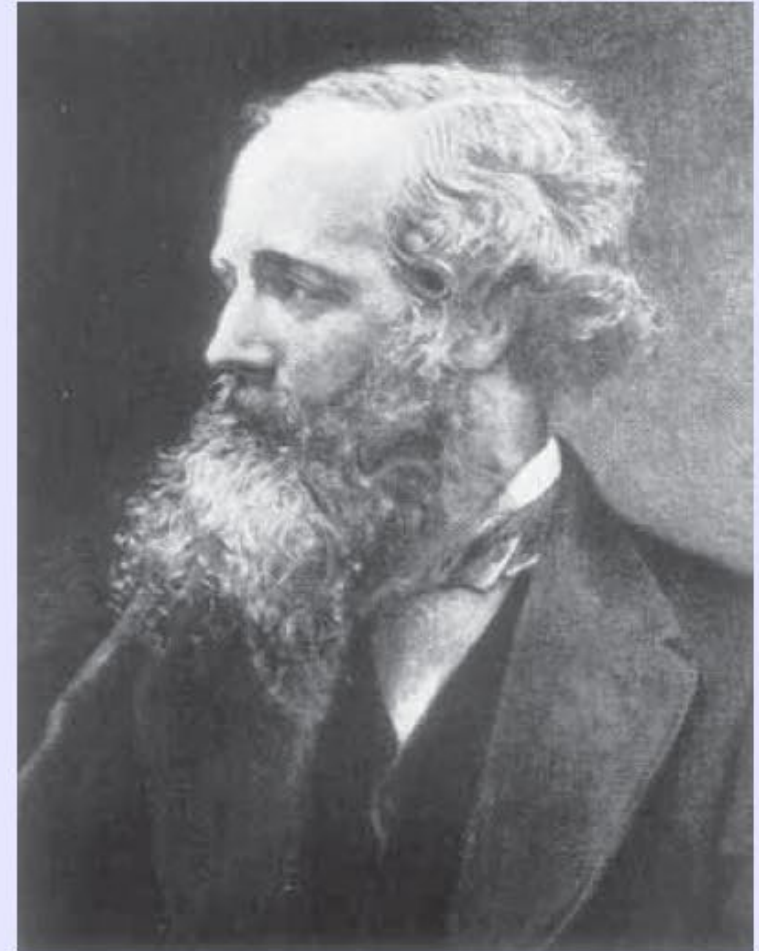


**solid-like  
creep**

**ramp stress history**

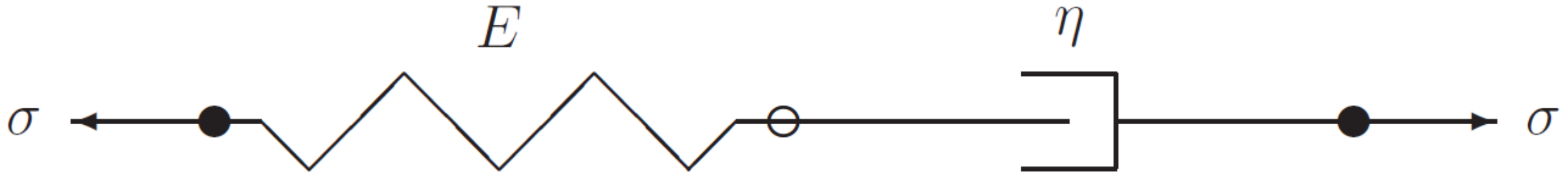
# Maxwell Model

*James Clerk Maxwell* [b. 13.6.1831 Edinburgh, Scotland, d. 5.11.1879 Cambridge, England] was Professor of Physics at various British Universities. He is best known for his unification of electricity, magnetism and light in the so-called Maxwell equations of electromagnetism that imply i.a. the finite speed of light, see his "A Treatise on Electricity and Magnetism" from 1873. As part of his occupation with the dynamical theory of gases he proposed in 1867 what is now called the Maxwell model for visco-elastic fluids.

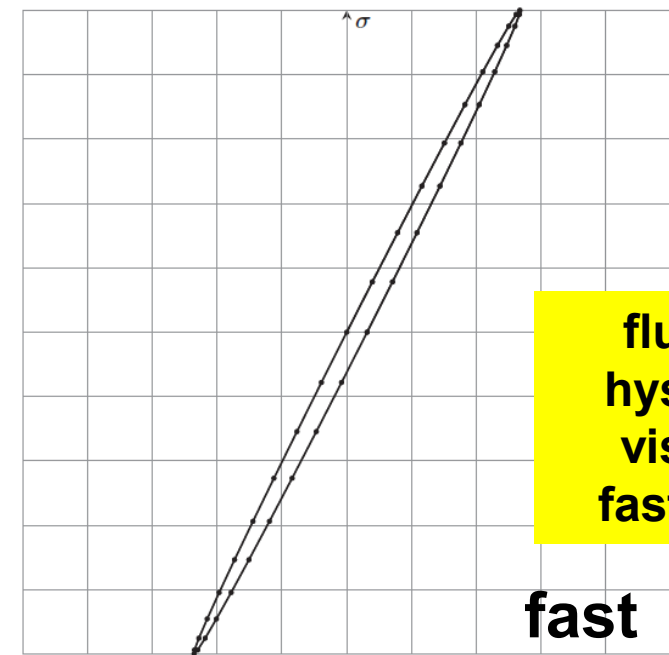
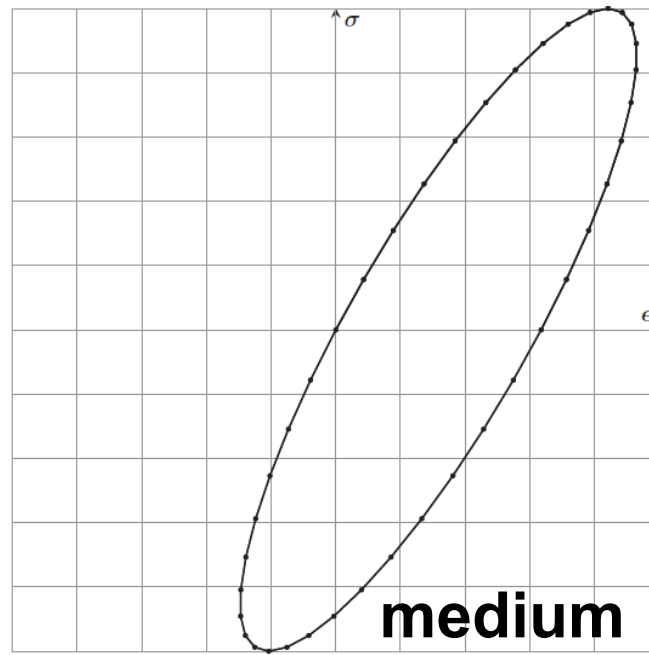
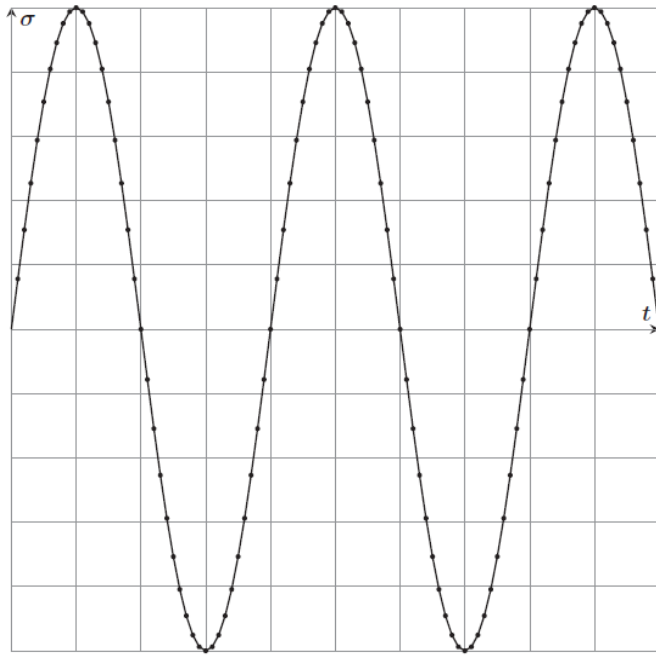




# Maxwell Model



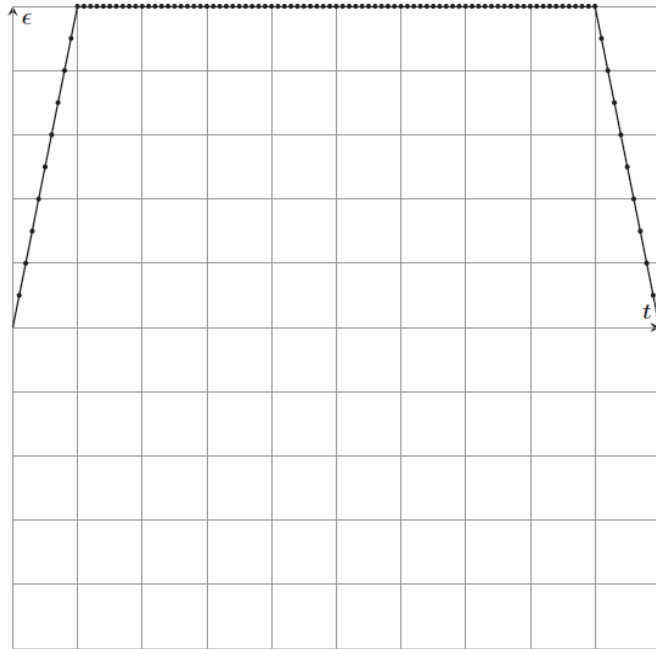
# Maxwell Model



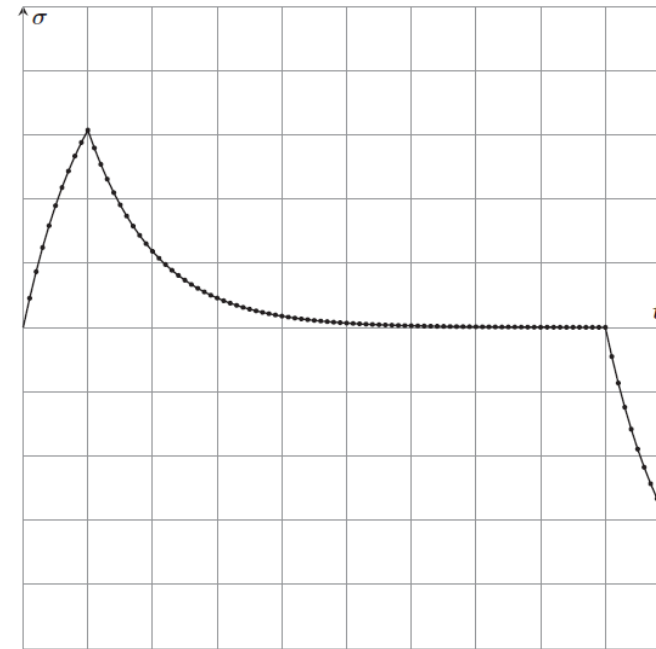
fluid-like  
hysteresis  
viscosity  
fast elastic

sine stress history

# Maxwell Model



ramp strain history



fluid-like  
relaxation

# Plasticity



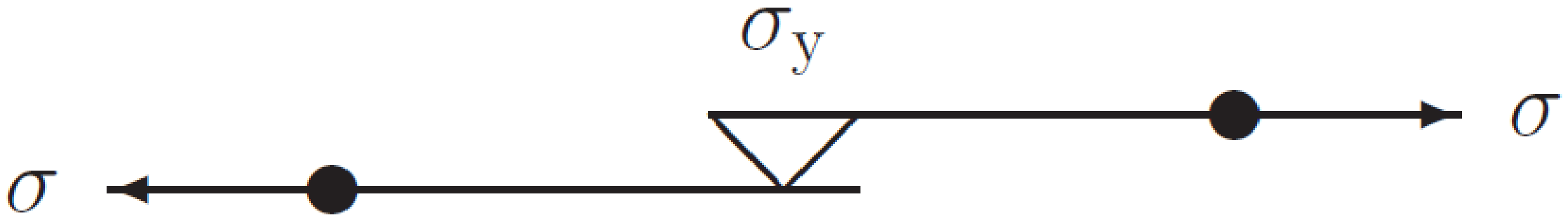


# Saint Venant Model

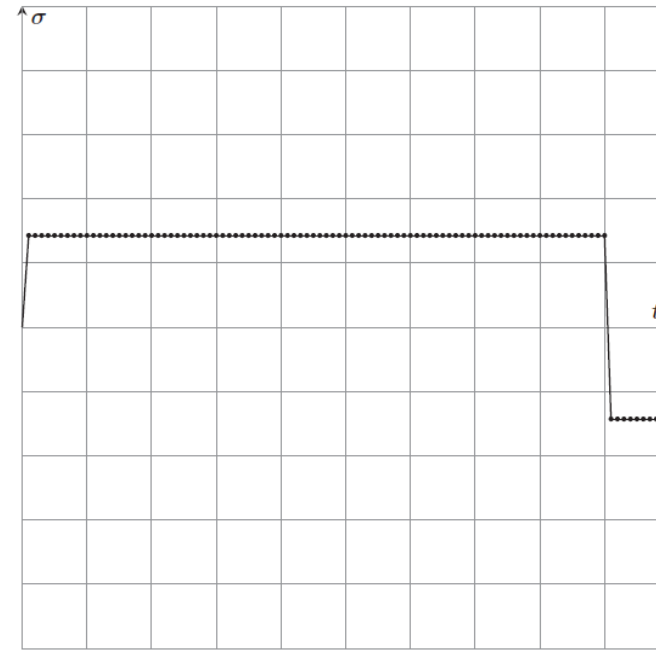
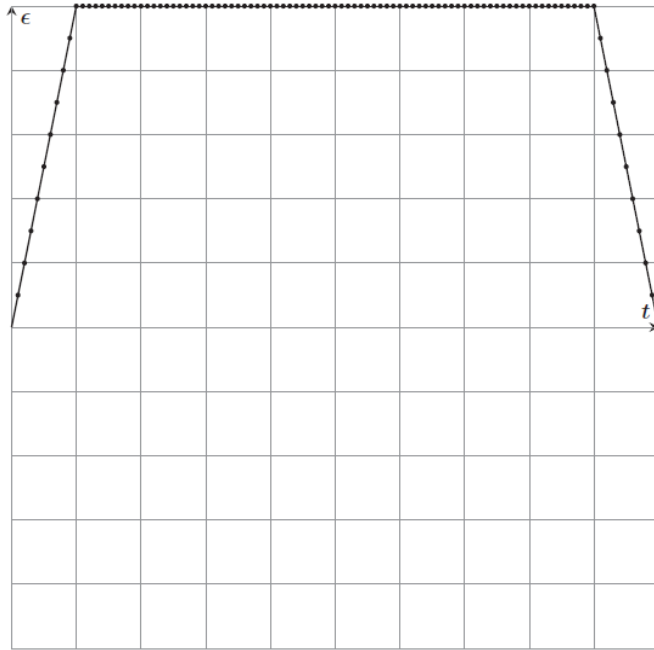
*Adhémar Jean Claude Barré de Saint-Venant* [b. 23.8.1797, Villiers-en-Bière, Seine-et-Marne, France, d. 6.1.1886, Saint-Ouen, Loir-et-Cher, France] worked initially as Engineer and later became Professor of Mathematics at the École des Ponts et Chaussées in Paris. He contributed largely to mechanics, elasticity, plasticity, hydrostatics and hydrodynamics. He was first to present a correct derivation of the Navier-Stokes equations in 1843. The St. Venant model for rigid-plastic solids is named after him.



# Saint Venant Model



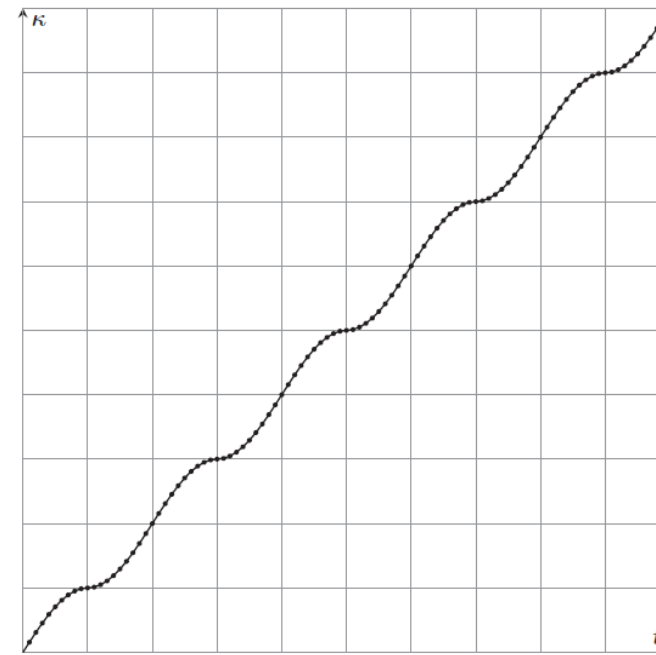
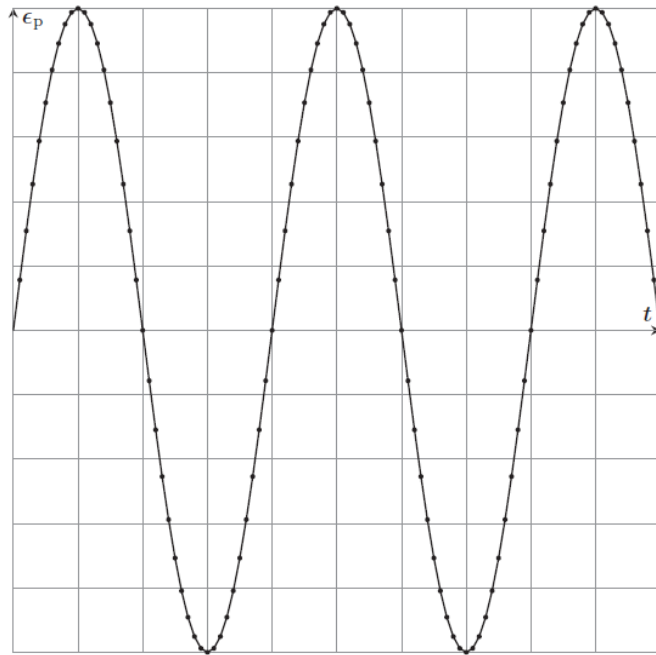
# Saint Venant Model



**yield stress**  
**rigid solid**  
**irreversibility**  
**no viscosity**

**ramp strain history**

# Saint Venant Model



**sine strain history**

**yield stress  
irreversibility  
internal var.  
no viscosity**

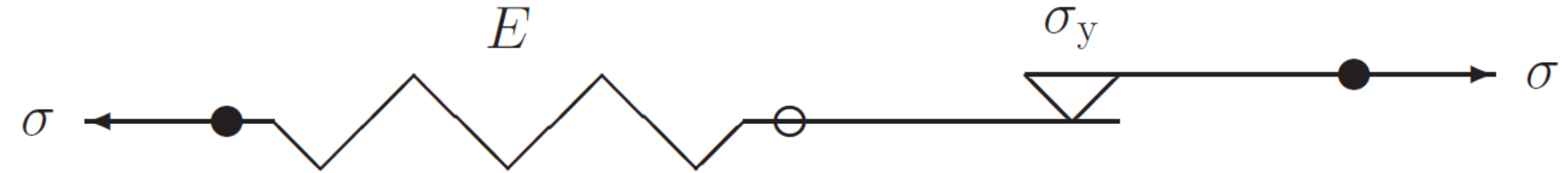


# Prandtl Model

*Ludwig Prandtl* [b. 4.2.1875, Freising, Germany, d. 15.8.1953, Göttingen, Germany] was Professor of Fluid Mechanics at the University Göttingen and the attached Kaiser Wilhelm Institute for Flow Research. He is the founding father of the boundary layer theory in fluid dynamics. Although mainly known for his research in aerodynamics, he also worked on problems of plasticity. His contribution to dry friction from 1928 gave an atomistic explanation of the static friction force. The Prandtl model thus describes solids displaying a yield stress after an initial elastic phase.

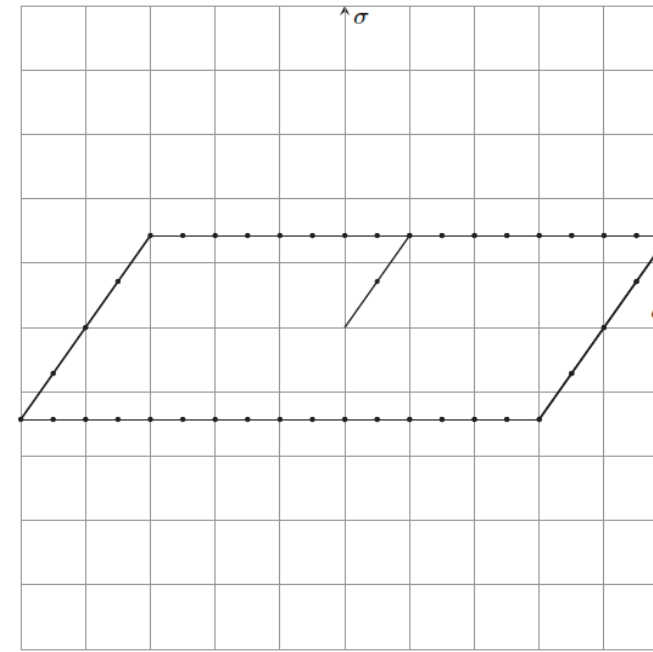
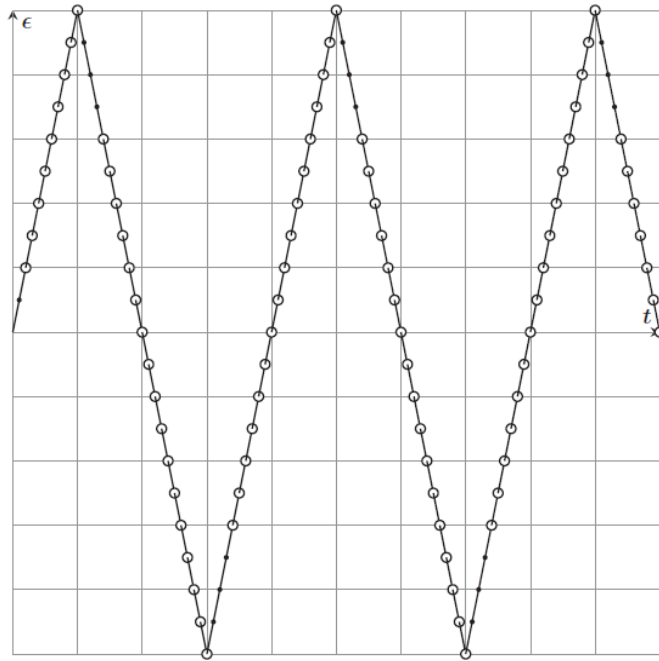


# Prandtl Model





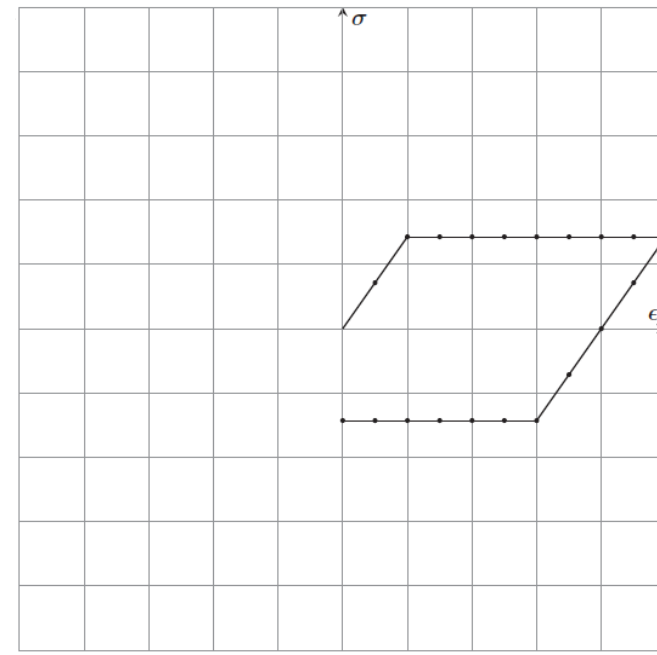
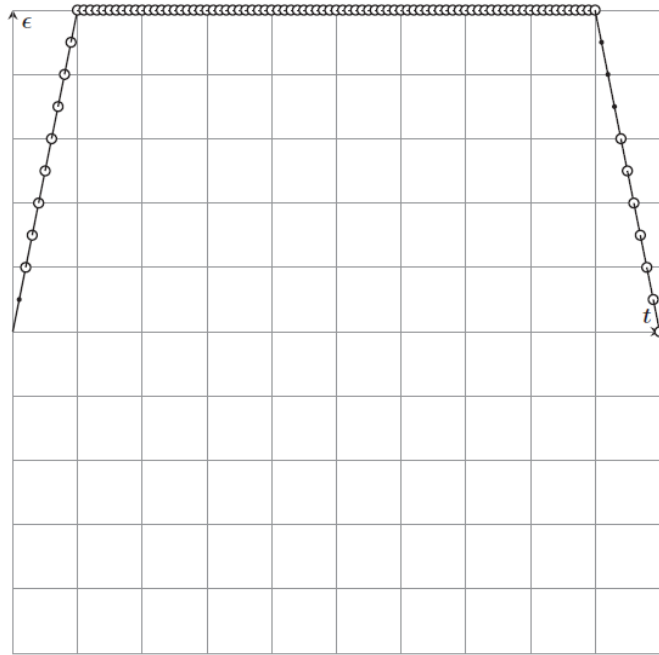
# Prandtl Model



**yield stress  
irreversibility  
no viscosity**

**zig-zag strain history**

# Prandtl Model



**yield stress  
irreversibility  
no viscosity**

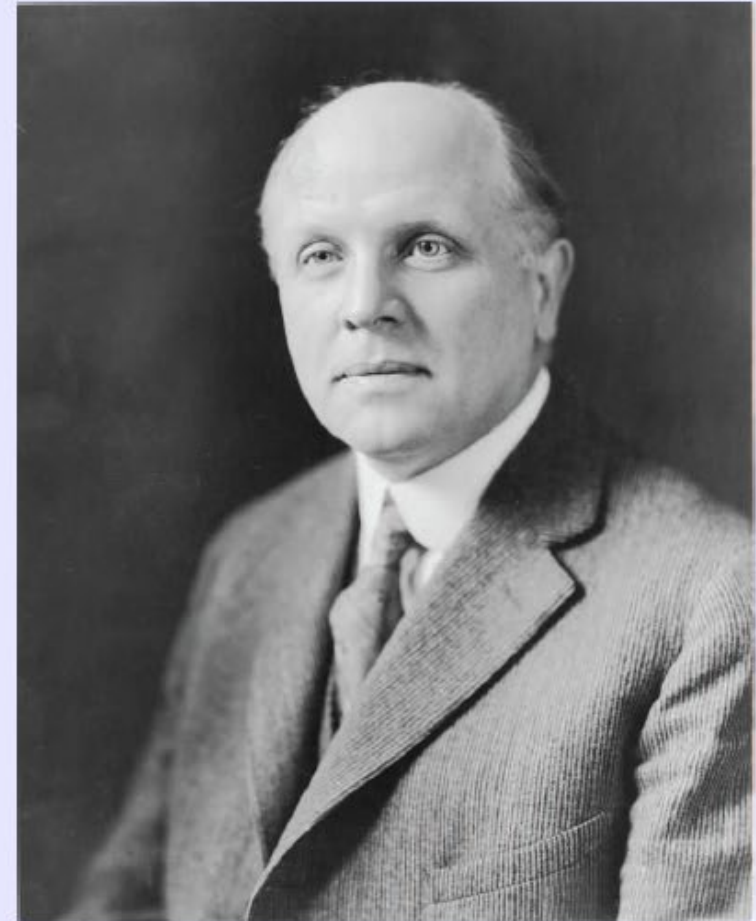
**ramp strain history**

# Visco-Plasticity

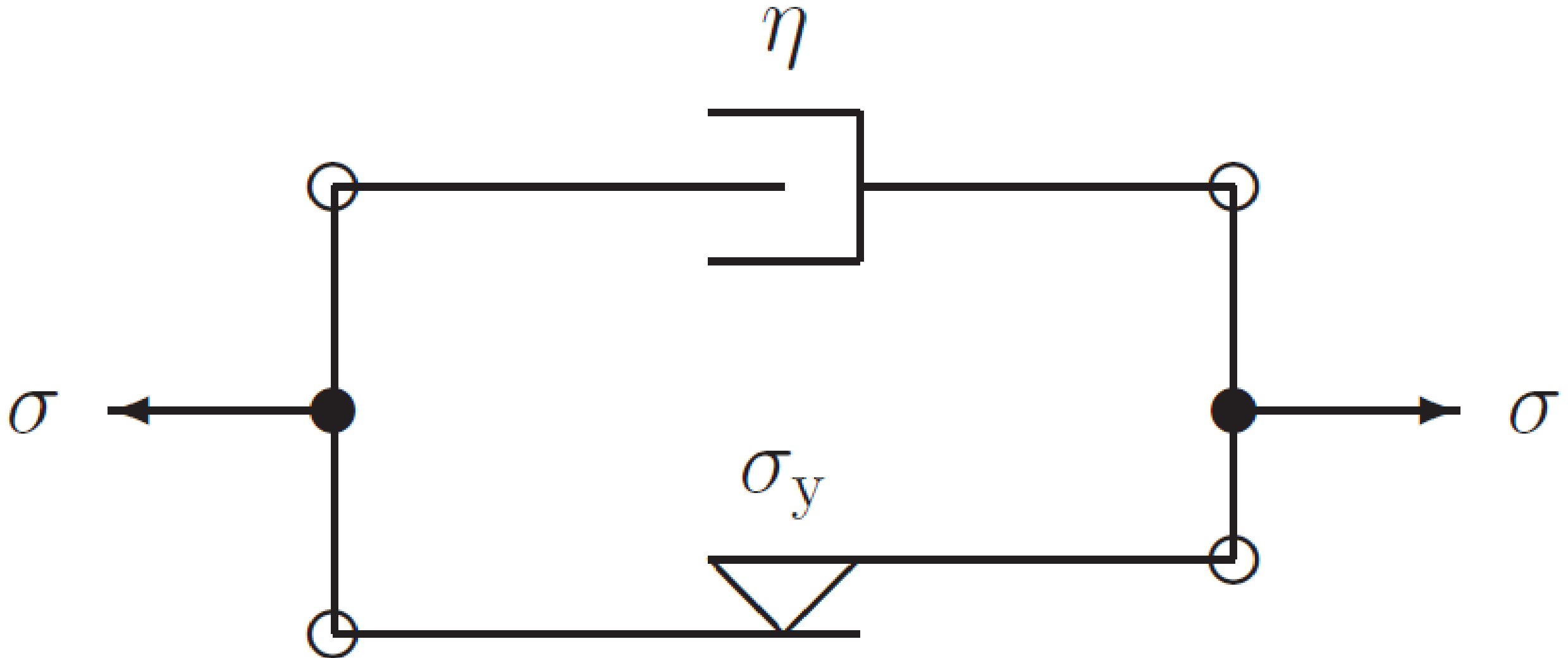


# Bingham Model

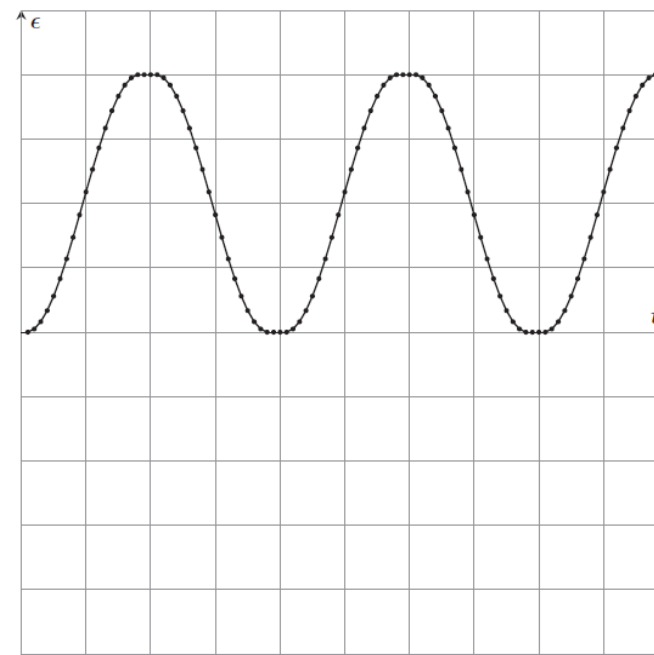
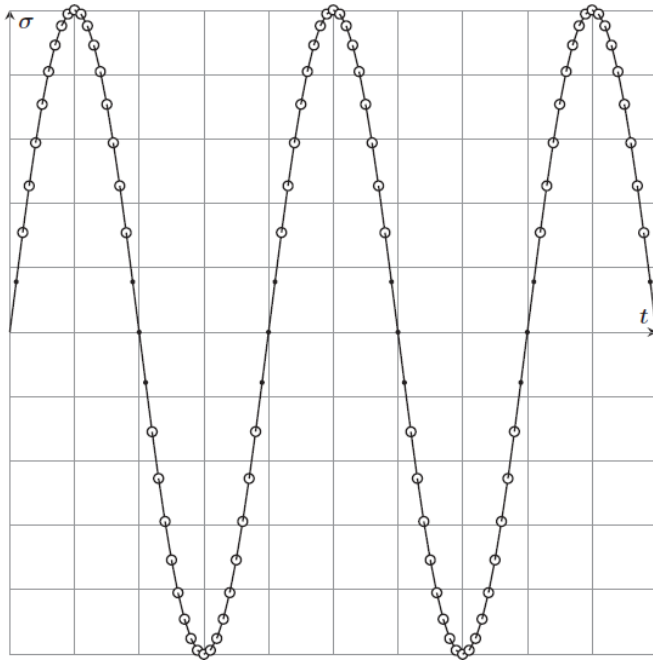
*Eugen Cook Bingham* [b. 8.12.1878, Cornwall, Vermont, USA, d. 6.11.1945, Easton, Pennsylvania, USA] was Professor of Chemistry at Lafayette College in Easton. He is the founding father (together with Markus Reiner) of the notion and science of rheology. Following a number of publications dealing with viscosity and plastic flow, he published his most popular treatise "Fluidity and Plasticity" in 1922. Fluids displaying a yield stress are named after him as Bingham fluids.



# Bingham Model



# Bingham Model

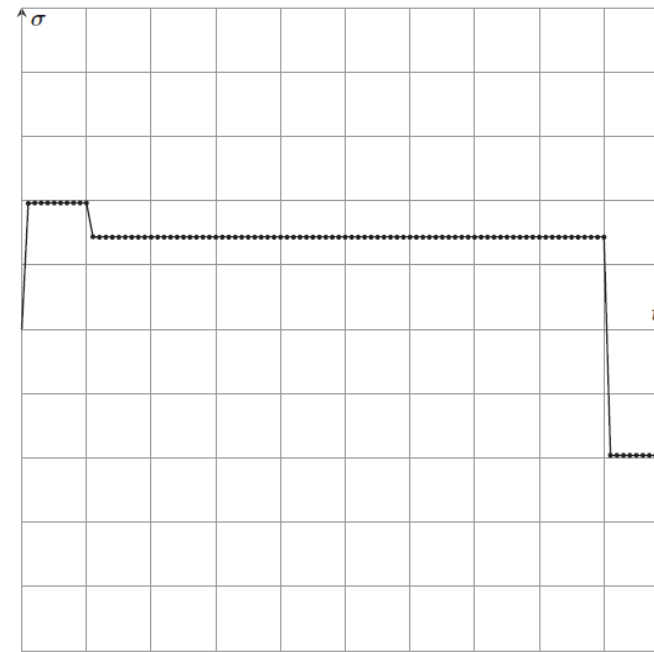
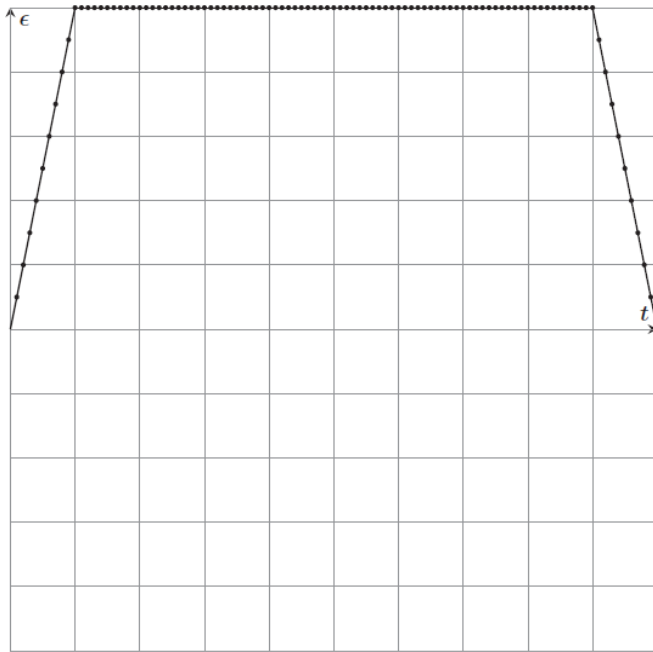


**rigid fluid  
phase shift  
viscosity**

**sine stress history**



# Bingham Model



**overstress**  
**irreversibility**  
**viscosity**

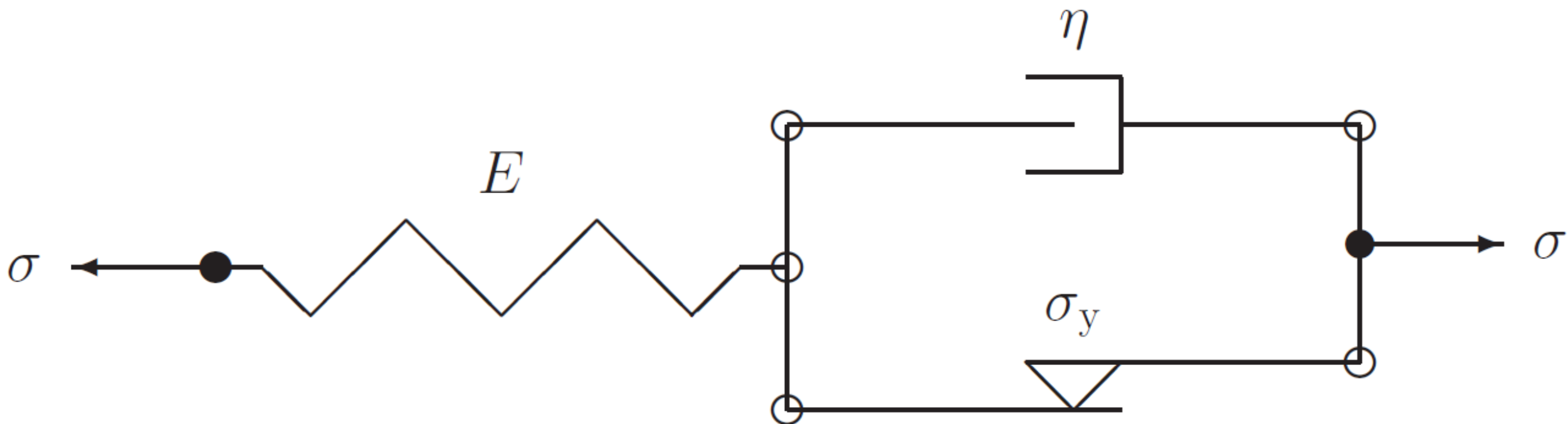
**ramp strain history**

# Perzyna Model

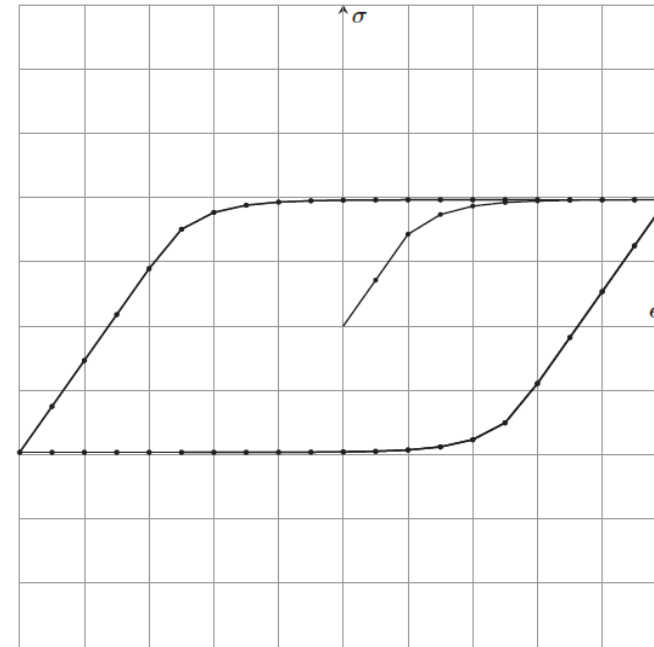
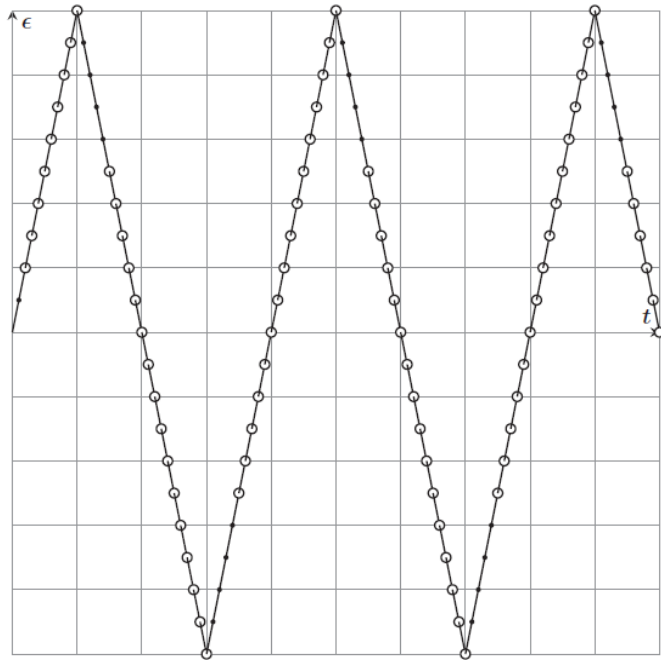
*Piotr Perzyna* [b. 1.8.1931, Niedźwiada, Poland, d. 22.6.2013, Warsaw, Poland] was Professor of Solid Mechanics at the Institute of Fundamental Technological Research, Polish Academy of Sciences (IPPT PAN) in Warsaw. He is the founding father of overstress-type visco-plasticity that is oftentimes named after him as Perzyna visco-plasticity. Since the foundation for Perzyna visco-plasticity was laid in his milestone publication from 1966, he continued to work on inelastic material models with internal variables.



# Perzyna Model



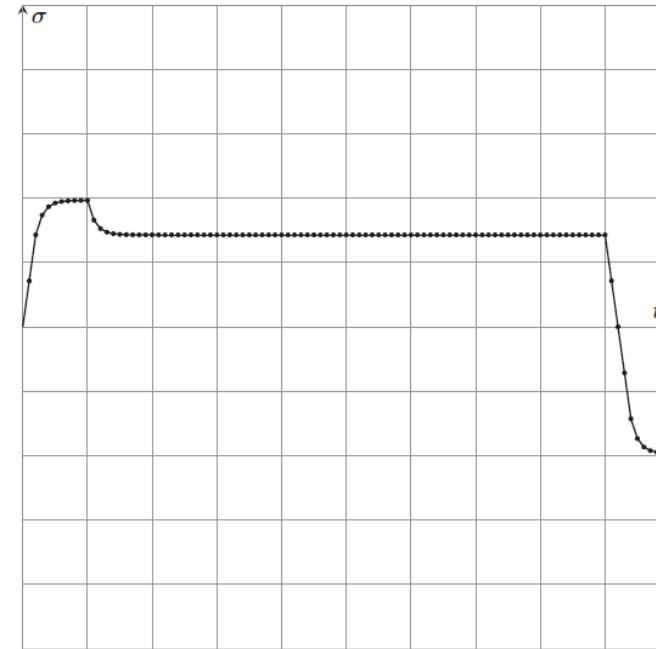
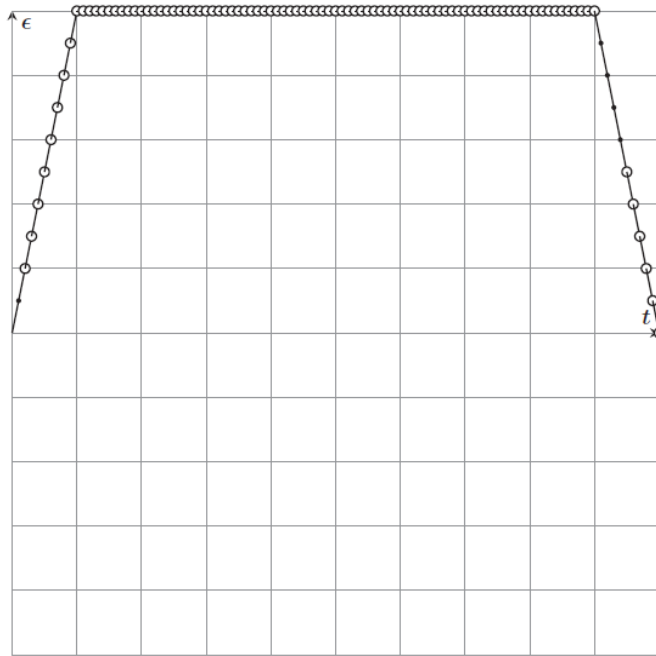
# Perzyna Model



**zig-zag strain history**

**overstress  
irreversibility  
viscosity**

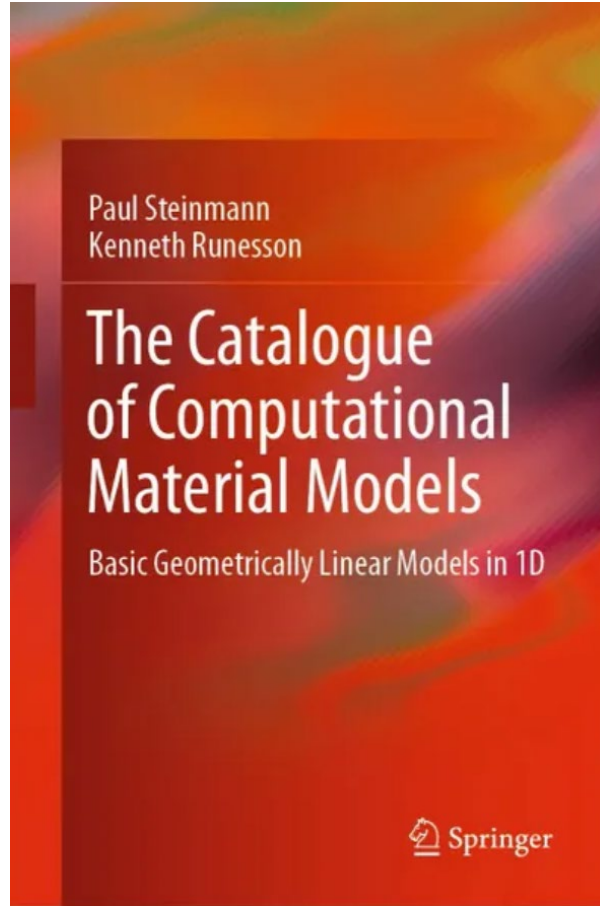
# Perzyna Model



**overstress**  
**irreversibility**  
**viscosity**

**ramp strain history**

# Everything You Always Wanted to Know about Modelling:



# ...but were Afraid to Ask